Applicability and Reliability of Spline Collocation Method for Third Order Boundary Value Problem

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Abstract

Here we solved the equation of Sandwich beam analysis problem. Solve the governing differential equation with using their associated boundary conditions with Bickley method. The beauty of this method is; we can solve higher order linear differential equation. We checked accuracy by using the analytical method also.

Keywords

Sandwich Beam problem, Differential equation, Spline collocation, linear equations, Hessenberg matrix.

1. Introduction

Sandwich constructions have two thin, elastic outer layers and a middle layer - core made of material with relatively small stiffness comparing to stiffness of the outer layers. Calculation of these constructions is based on the supposition that all three layers deform simultaneously, and as the result a unique neutral line is formed between the outer layers. Beams formed by a few laminae of differential materials are known as sandwich beams. In an analysis of such beams subject to uniformly distributed load along the entire length, Krajcinvic [1] found that the distribution of shear deformation ψ is governed by the linear ordinary differential equation.

2 Governing equations

Here we find shear deformation ψ of sandwich beam analysis problem found by Krajcinvic [1] in term of linear ordinary differential equation is as follows;

$$\frac{d^3\Psi}{dx^3} - k^2\frac{d\Psi}{dx} + a = 0 \qquad (1)$$

Where k^2 and a are physical constants which depend on the elastic properties of lamina.

For the free ends, the condition of zero shear bimoment at both ends leads to the boundary conditions

$$\frac{d\Psi(0)}{dx} = \frac{d\Psi(1)}{dx} = 0 \qquad (2)$$

For symmetry conditions,

$$\Psi\left(\frac{1}{2}\right) = 0 \qquad (3)$$

The spline collocation method is used to solve the equation (1) with help of boundary conditions (2 & 3).

3 Solution of sandwich beam problem by using Bickley's method.

Let us consider the quartic spline to solve third order boundary value problem.

The quartic spline is defined by [3]

$$s(x) = a_0 + b_0(x-x_0) + \frac{1}{2}c_0(x-x_0)^2 + \frac{1}{6}d_0(x-x_0)^3 + \frac{1}{24}\sum_{k=0}^{n-1} e_k(x-x_k)_+^4 \quad (4)$$

Substitute (4) and its respective derivatives in (1), collocations can be obtained as follows

$$\sum_{k=0}^{n-1} e_k[(x_i-x_k) - \frac{k^2}{6}(x_i-x_k)^3] + d_0[1-\frac{k^2}{2}(x_i-x_0)^2] + c_0[-k^2(x_i-x_0)] + b_0[-k^2] = -a \quad (5)$$

For $k=5$, $a=1$ and different values of i in (5) and by using the boundary conditions in (4), different linear equations are found and their solutions are as follows:

Table 1: Solution of linear differential equation using Spline Collocation Method.

X	Spline solution Ψ(x) For h=0.1	x	Spline solution Ψ(x) For h=0.1
0.0	-0.0121	0.6	0.0034
0.1	-0.0113	0.7	0.0066
0.2	-0.0092	0.7	0.0094
0.3	-0.0064	0.9	0.0114
0.4	-0.0033	1.0	0.0122
0.5	0.0000		

Thus, a method can successfully apply to linear boundary value problem.

4 Solution of sandwich beam problem by using Analytical method.

Now aim is to find the analytic solution of given problem and do the comparison of spline solution with analytical solution.

Try to find the solution directly analytical way.

Auxiliary equation (1) as $D^3 - 25D + 1 = 0$ (6)

Where $k=5$ and $a=1$.

Solutions of this cubic equation (6) is

Thus $\Psi(x) = -\frac{e^{-5x}}{5}c_1 + \frac{e^{5x}}{5}c_2 + c_3 + \frac{x}{25}$ (7)

and $\Psi'(x) = e^{-5x}c_1 + e^{5x}c_2 + \frac{1}{25}$ (8)

Use boundary conditions (2 & 3) to find constants in (7).

Final solution of (7) is

$$\Psi(x) = 0.0397\frac{e^{-5x}}{5} - 0.0003\frac{e^{5x}}{5}c_2 - 0.0200 + \frac{x}{25}$$

Analytic solution of equation (1) as follows:

Table 2: Comparison of Spline Collocation Method and Analytical method.

X	Spline solution $\Psi(x)$ For h=0.1	Analytic solution $\Psi(x)$ For h=0.1
0.0	-0.0121	-0.0121
0.1	-0.0113	-0.0113
0.2	-0.0092	-0.0092
0.3	-0.0064	-0.0065
0.4	-0.0033	-0.0034
0.5	0.0000	0.0000
0.6	0.0034	0.0032
0.7	0.0066	0.0063
0.8	0.0091	0.0089
0.9	0.0109	0.0107
1.0	0.0114	0.0111

Thus, from the table; it can be concluding that spline method also gives nearest and accurate solution of give problem. Thus it can be applied any of the higher order differential equation problem.

5. Conclusion

A special emphasize is given to the applicability and reliability of the method of spline collocation, to see this, the spline functions were used to interpolate the solutions to various types of differential equations occurring in the study of several physical phenomena in engineering sciences. In all, it is worthy to mention that the spline collocation technique with the above approach is helpful to solve various types of fluid flows problems. According to the experience during the research work, it can be concluded that, cubic splines are better for the second order boundary value problems, quartic splines are better for the third ordered boundary value problems and the quintic splines are better for the fourth ordered boundary value problems.

References:

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